to allow the built-in cross section to warp because the warping can be considered the natural response of the beam to the end moment for holding the neutral axis in the level position inside the built-in cross section.

## Axial Instability

Hodges' argument regarding the axial instability can also be applied to the transverse instability as well. As was discussed in Ref. 1, the maximum stress in a rotating steel or aluminum beam normally exceeds its yield strength at a rotating speed much lower than any of the unstable rotating speeds for these two instabilities. Similar to Brunelle's analysis,[10] the discussion of inertioelastic instabilities in Ref. 1 was intended only to show a mathematical prediction based on the linear theory rather than to indicate instabilities that actually exist in rotating metallic beams.

## Dissipative Forces

Regarding whether or not the variational principle can be applied to cases with nonconservative forces, Goldstein[11] provided some history of this subject. For a system with only dissipative forces and no potential function, the Lagrangian was expressed by Goldstein[11] as

$$L = T - U \tag{1}$$

where $U$ was called a "general potential" or a "velocity-dependent potential" by Goldstein.[11] According to Goldstein,[11] the German mathematician Schering seems to have been the first to seriously attempt to include velocity-dependent forces in the framework of mechanics in 1873. Goldstein[11] suggested the name "generalized potential" to include ordinary potential energy within this designation. The time integration of the virtual work expressed in Ref. 1 actually is a result of considering such a generalized potential in the variational principle.

The dissipative forces considered in Ref. 1 were viscous damping forces in the structure. Terms involving $\Omega$ in Eq. (36) did not appear in the governing equations, Eqs. (40–42). Since these governing equations were derived to include both the steady-state analysis and the vibration analysis, general expressions of the velocity components of particles in the structure were used in Eq. (36) to indicate the damping force is being related to the particle velocity. To be more precise, for the case of describing vibration only, these terms should have been left out in Eq. (36). However, excluding them does not change the governing equations, Eqs. (40–42), shown in Ref. 1.

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# Comment on "Application of Singular Value Decomposition to Direct Matrix Update Method"

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IN a recent Technical Note by To,[1] he investigated the mass matrix correction method of Ref. 2. His conclusions may be briefly summarized as follows: the matrix representing the change in the mass will have a rank equal to the lower of the number of modes used or the number of mass error sites. I find no fault with the analysis as presented and I am, in fact, pleased with the results of this study. I believe, however, that I should comment on some common misconceptions of the applicability of model updating methods which are in this paper[1] and in many others.

In studies of such techniques, as in Ref. 1, it is common to treat the structure as a finite set of lumped masses connected by linear springs. It is then possible to consider complete and incomplete sets of measured modes. This concept was also implied by myself in Ref. 2. When the method was applied to a real problem[3] and enhanced by other relevant techniques, such as, from Ref. 4, this was found to be an inappropriate oversimplification. This and other related considerations are discussed in some detail in Refs. 5 and 6. A very brief summary of this important, but rarely acknowledged, problem follows.

A real structure has essentially an infinite number of degrees of freedom. A complete finite element representation of a linear real structure must be nonlinear. For a real structure, there are an infinite number of "good" linear models having a finite number of degrees of freedom.[6] Each of these models has a limited range of applicability. Only a subset of the modes of the model will correspond to modes of the structure. Thus, the concept that one needs "all" the modes for a procedure to give the exact answer makes no sense at all.

The "improved" mass matrix of Refs. 2 and 3 is not a "correct" matrix. It is one of the many which will yield an analytical model which will behave like the structure within the limited applicable range. Since it also deviates by some minimum amount from a "pretty good" analytically derived model, one may expect that it also has some applicability over a somewhat wider range than the test data. Further studies are required in this area.

To[1] uses as his structure a finite system with a finite number of modes. He demonstrates, that for this system, the method of Ref. 2 will work when using less than all the modes, if the number of modes is at least equal to the number of mass error sites. Although his conclusion is appropriate for the problem he presented, unfortunately, it does not apply to the problem of improving an analytical model of a physical structure.

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